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CONJUGATED HEAT TRANSFER FROM CIRCULAR CYLINDERS IN LOW REYNOLDS NUMBER FLOW

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Abstract — The problem of conjugated heat transfer from a circular cylinder with a heated core region in low Reynolds number flow is considered. The solid may consist of several layers with different thermal conductivities. The energy equations of the fluid and the solid are solved under the conditions of equality in heat flux and temperature at the fluid–solid interface and the solid–solid interfaces. Conditions at infinity are applied for the flow field by Imai's asymptotic solution and for the temperature field by a matching to the corresponding asymptotic temperature solution. Results are given for various solid materials and fluids.

NOMENCLATURE

a, cylinder radius;
A_n, constant;
C_D, drag coefficient;
D, diameter of the cylinder;
D_c, diameter of the core region;
D_i, outer diameter of the ith solid layer;
erf, error function;
F, function in the asymptotic solution of the temperature field;
H, step size in the numerical calculations;
h, film heat transfer coefficient;
K_n, modified Bessel function of the second kind;
k_f, thermal conductivity of the fluid;
k_{s_i}, thermal conductivity of the ith solid layer;
k_s, thermal conductivity of the solid when only one solid layer exists;
Nu_D, Nusselt number ($= hD/k_f$);
Nu_m, mean Nusselt number;
n, integer;
P, function in the asymptotic solution of the flow field;
Pr, Prandtl number;
Q, function in the asymptotic solution of the flow field;
R, optimal relaxation parameter of the heat conduction equation;
Re_D, Reynolds number ($= U_\infty D/\nu$);
r, radial coordinate in the polar coordinate system (r, θ);
T, dimensionless temperature;
t, temperature;
t_c, temperature of the core region;
t_i, temperature within the ith solid layer;
t_∞, temperature of the fluid far away from the cylinder;
U_∞, velocity of the fluid far away from the cylinder;
X, variable in the asymptotic solution of the temperature field;
x, y, rectilinear coordinate system.

Greek symbols
Δ, Laplacian operator;
η, dimensionless coordinate in the (ξ, η)-coordinate system;
θ, polar angle in the polar coordinate system (r, θ);
ν, kinematic viscosity of the fluid;
ξ, dimensionless coordinate in the (ξ, η)-coordinate system;
φ, angle measured from the forward stagnation point;
Ψ, streamfunction;
Ψ', dimensionless streamfunction;
ω, vorticity;
ω', dimensionless vorticity ($= \omega a/U_\infty$).

INTRODUCTION

WHEN a solid body of finite dimensions and moderate thermal conductivity is adjacent to a fluid flow domain, the temperature field in the fluid is coupled with the temperature distribution in the solid. It is then necessary to solve the energy equations for the fluid and the solid body under the conditions of equality in temperature and heat flux at the interface. This presents a conjugate heat transfer problem.

Problems of this kind are complicated but have been increasingly studied during recent years. Both analytical and numerical methods have been used. Some of the papers will be mentioned below. Perelman [1] studied the problem of slip flow past a semi-infinite solid having distributed heat sources. An analytical solution was obtained for the integral equation arising from coupling the Laplace transform solution for the fluid and the Fourier transform solution for the solid. He also considered the problem of transport to a laminar boundary layer.

Heat transfer to a slug flow past a flat plate was treated by Sell and Hudson [2]. They obtained the temperature distribution in the flat plate and the fluid in terms of a series expansion. Conjugated heat transfer has been treated for circular tubes in [3–7] and for parallel plates in [3, 8–10]. Luikov *et al.* [11] studied the conjugated heat transfer from a flat plate in a compressible gas flow by the method of separation of variables. Perelman *et al.* [12] considered the unsteady conjugated heat transfer problem in a semi-infinite plate with heat sources. They also used the separation of variables technique.

Convective heat transfer for steady and unsteady regimes when the unsteady state is the result of heat sources present in the body, was investigated by Luikov *et al.* [13]. They obtained analytical solutions for steady state heat transfer of a thin-wall wedge and for unsteady state heat transfer of a thin plate normal to an incident viscous incompressible flow.

Luikov [14] analysed the conjugated heat transfer from a flat plate of finite thickness in a laminar incompressible flow. Design formulas for the local Nusselt number were suggested. This analysis was extended by Payvar [15], thereby using Lighthill's method of analysis for laminar boundary layer heat transfer problems.

Sohal and Howell [16] considered the determination of the temperature of a flat plate in the case of combined conduction, convection and radiation.

The coupled heat transfer problem of two laminar free convection systems with a conducting vertical plate in between was studied by Lock and Ko [17].

Karvinen [18, 19] presented an approximate method for calculation of conjugated heat transfer in a flat plate by using analytical solutions for the convective heat transfer field and finite-difference solutions for the heat conduction.

Olsson [20, 21] studied the conjugated heat transfer for a wedge in laminar flow by using an integral method and an extended Blasius technique. A similar study was performed for a lenticular cylinder by Halse [22].

Torkelsson [23] considered the conjugated heat transfer for a wedge in laminar as well as turbulent flow. The method of finite differences and the finite element method were used.

Recently, Patankar [24] presented a method (for finite difference solutions) to account for discontinuities in thermal conductivity and other transport properties. He showed that it is more appropriate and more accurate to calculate the heat flux across an interface from the harmonic mean of the conductivities on the two sides of the interface, than from the arithmetic mean. As examples Patankar gave the flow in a duct with an internal fin and the fully developed heat transfer in a square duct with finite wall thickness.

In the present work, the conjugated heat transfer from a circular cylinder with a heated internal core region in low Reynolds number flow is considered. One objective of the paper is to present the influence of the thermal conductivity ratio (solid to fluid) on the heat transfer. Another objective of the paper is to perform accurate flow and temperature calculations by using accurate far field representations of both the flow and temperature fields.

The common case with a uniform temperature prescribed at the solid–fluid interface is obtained as the limit of an infinite ratio of the thermal conductivity of the solid to that of the fluid.

Studies of the flow field around a circular cylinder for low Reynolds numbers have been made by many investigators, for instance Thom [25, 26], Takami and Keller [27], Kawaguti and Jain [28], and Nieuwstadt and Keller [29]. A review can be found in [30]. The technique to calculate the flow field can therefore be said to be known.

For the heat transfer field only pure convection, that is the cases of uniform temperature and uniform heat flux at the solid–fluid interface, have been treated. See [31–35].

For the problem of coupled heat conduction within a solid circular cylinder to forced convection in a fluid (conjugated heat transfer), no study seems to have been performed.

PROBLEM UNDER CONSIDERATION

We are considering a long horizontal circular cylinder which is aligned normal to a uniform, undisturbed oncoming free stream with velocity U_∞ and temperature t_∞. The diameter of the cylinder is D. The cylinder has a heated core region with a diameter D_c and a temperature t_c. The cylinder material may consist of concentric layers with different thermal conductivities k_{s_1}, k_{s_2}, etc. The thermal conductivity of the fluid is k_f. See Fig. 1.

We are interested in the local and mean heat transfer coefficients Nu_D, and how they are influenced by the conductivity ratios k_{s_1}/k_f, k_{s_2}/k_f, etc. D_c/D, and the Reynolds number Re_D. Of special importance is the temperature distributions at the interface between the fluid and the solid, and at the interfaces between the heat conducting solid layers.

The physical properties of the fluid and the solid layers are assumed to be constant. Thus it is required that the temperature differences are small.

The study has been performed for Reynolds numbers in the range of $5 \le Re_D \le 40$ thereby using the further assumptions of steady state and symmetry around the stagnation lines which might be good assumptions according to experiments. See [36] and [37].

The ratio of the Grashof number to the square of the Reynolds number is assumed to be much smaller than unity and thus the influence of natural convection is neglected.

Since the physical properties are assumed to be constant and thereby independent of the temperature, the flow field is not coupled with the temperature field.

The problem is of interest in hot-wire anemometry, heat exchanger design and the rating of electrical conductors.

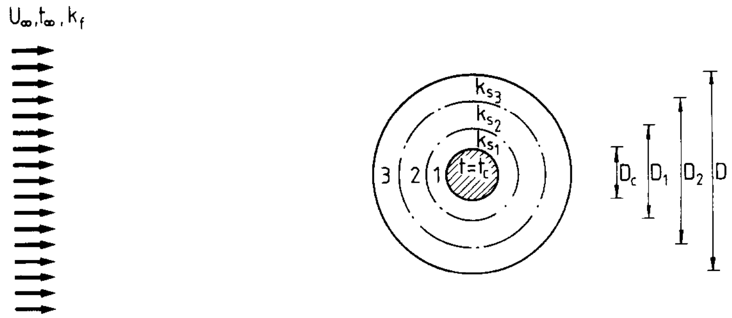

FIG. 1. Problem under consideration.

BASIC EQUATIONS

With the assumptions above, the basic equations are:

Flow field

$$\nu \Delta\omega = \frac{\partial(\omega, \Psi)}{\partial(x, y)}, \tag{1}$$

$$\Delta\Psi = -\omega. \tag{2}$$

Convective heat transfer field

$$\Delta t = \frac{Pr}{\nu} \frac{\partial(t, \Psi)}{\partial(x, y)}. \tag{3}$$

Heat conduction within any solid layer

$$\Delta t = 0 \tag{4}$$

where Ψ is the streamfunction, ω the vorticity, ν the kinematic viscosity, t the temperature, and Pr the Prandtl-number of the fluid. x, y are the independent Cartesian coordinates. Δ is the Laplacian operator.

BOUNDARY CONDITIONS

Equations (1)–(4) are to be solved with respect to some boundary conditions. We use the following boundary conditions.

Within the core region:

$$x^2 + y^2 \leq D_c^2/4: \quad t = t_c. \tag{5}$$

At the ith solid–solid interface (between layers i and $i+1$):

$$x^2 + y^2 = D_i^2/4, \text{ all } \theta: \quad t_i = t_{i+1}, \tag{6}$$

$$-k_{s_i}\left(\frac{\partial t}{\partial r}\right)_i = -k_{s_{i+1}}\left(\frac{\partial t}{\partial r}\right)_{i+1}. \tag{7}$$

At the fluid–solid interface:

$$x^2 + y^2 = D^2/4, \text{ all } \theta: \quad t_{\text{convection}} = t_{n_{\text{conduction}}}, \tag{8}$$

$$-k_f\left(\frac{\partial t}{\partial r}\right)_{\text{convection}} = -k_{s_n}\left(\frac{\partial t}{\partial r}\right)_{\text{conduction}}. \tag{9}$$

r, θ are here the polar coordinates. Subscript n denotes the solid layer adjacent to the fluid.

$$\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial y} = 0. \tag{10}$$

Far away from the cylinder:

$$x^2 + y^2 \to \infty: \quad t \to t_\infty, \tag{11}$$

$$\Psi \to U_\infty y, \tag{12}$$

$$\omega \to 0. \tag{13}$$

Equations (6) and (7), and (8) and (9) express the thermal conditions at the solid–solid interfaces and at the fluid–solid interface, respectively. Through these conditions, continuity in temperature and heat flux, the temperature fields in the solid layers are coupled with the temperature in the fluid.

TRANSFORMATION OF THE BASIC EQUATIONS AND THE BOUNDARY CONDITIONS

For accuracy in the numerical treatment, new rectangular coordinates (ξ, η) are introduced by the conformal transformation

$$\xi = \frac{1}{\pi}\ln(r/a), \tag{14}$$

$$\eta = 1 - \theta/\pi. \tag{15}$$

(See Fig. 2, $a = D/2$.)

The following non-dimensional variables are introduced:

$$\Psi' = \Psi/aU_\infty; \quad \omega' = \omega a/U_\infty; \quad T = (t - t_\infty)/(t_c - t_\infty).$$

By using equations (14) and (15), and the non-dimensional variables above, the basic equations now read:

$$\Delta\omega' = \frac{Re_D}{2} \frac{\partial(\Psi', \omega')}{\partial(\xi, \eta)}, \tag{16}$$

$$\Delta\Psi' = -\pi^2\omega' e^{2\pi\xi}, \tag{17}$$

$$\Delta T = \frac{Re_D Pr}{2} \frac{\partial(\Psi', T)}{\partial(\xi, \eta)}, \tag{18}$$

$$\Delta T = 0. \tag{19}$$

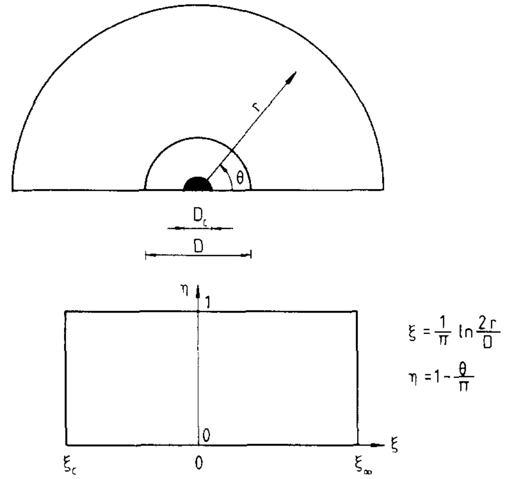

FIG. 2. Coordinate transformation and the calculation domain for the heat transfer field.

In equations (16)–(19) Δ is $(\partial^2/\partial\xi^2) + (\partial^2/\partial\eta^2)$.

The boundary conditions are transformed into:

$$\xi \le \xi_c: \quad T = 1. \tag{20}$$

$$\xi_i = \frac{1}{\pi}\ln\frac{D_i}{D}, \text{ all } \eta: \qquad T_i = T_{i+1}. \tag{21}$$

$$-k_{s_i}\left(\frac{\partial T}{\partial \xi}\right)_i = -k_{s_{i+1}}\left(\frac{\partial T}{\partial \xi}\right)_{i+1}. \tag{22}$$

$$\xi = 0, \text{all } \eta: \qquad T_{\text{convection}} = T_{n_{\text{conduction}}} \tag{23}$$

$$-k_f\left(\frac{\partial T}{\partial \xi}\right)_{\text{convection}} = -k_{s_n}\left(\frac{\partial T}{\partial \xi}\right)_{n_{\text{conduction}}}. \tag{24}$$

$$\frac{\partial \Psi'}{\partial \xi} = \frac{\partial \Psi'}{\partial \eta} = 0 \tag{25}$$

$$\xi \to \infty: \quad \Psi' \to e^{\pi\xi}\sin\pi(1-\eta) \tag{26}$$

$$\omega' \to 0 \tag{27}$$

$$T \to 0 \tag{28}$$

Along the stagnation lines, we have due to symmetry:

$$\begin{matrix}\eta = 0: \\ \eta = 1:\end{matrix} \quad \frac{\partial \Psi'}{\partial \xi} = 0, \quad \omega' = 0, \quad \frac{\partial T}{\partial \eta} = 0. \tag{29}$$

CORRECTION OF THE BOUNDARY CONDITIONS AT INFINITY

For a practical reason, the calculation domain must be cut off at some finite distance from the cylinder. The assumption of the uniform stream boundary conditions [equations (26)–(28)] at finite distances from the cylinder is not so very satisfactory because of the slow decay of the flow in the wake. By using equations (26)–(28), the drag coefficient is found to vary with ξ_∞, and the temperature field becomes inaccurate. In order to obtain accurate flow and temperature fields it is necessary to correct these boundary conditions. For the streamfunction and the vorticity, the asymptotic solution deduced by Imai [38] can be used. Here Imai's solutions including two terms are employed. We then have:

$$\Psi'(\xi_\infty,\eta) = e^{\pi\xi_\infty}\sin\pi(1-\eta) + \frac{C_D}{2}\{(1-\eta) - \operatorname{erf}(Q)\}$$
$$- \frac{C_D^2 Re_D}{16\sqrt{\pi}}\left[\frac{1}{P}\{\sqrt{2}(\operatorname{erf}(\sqrt{2}Q) - 1) - e^{-Q^2}\operatorname{erf}(Q)\} + \frac{\sqrt{2}P}{P^2+Q^2}\right], \tag{30}$$

$$\omega'(\xi_\infty,\eta) = -\frac{C_D Re_D}{4\sqrt{\pi}}\frac{Q}{e^{\pi\xi_\infty}}e^{-Q^2} + \frac{C_D^2 Re_D^2}{64\sqrt{\pi}\,e^{\pi\xi_\infty}}$$
$$\times\left[-\frac{1}{P}e^{-Q^2}\left\{\frac{2}{\sqrt{\pi}}Q\,e^{-Q^2} + (2Q^2-1)\operatorname{erf}(Q)\right\} + \frac{1}{P^3}\{\sqrt{2}(\operatorname{erf}(\sqrt{2}Q) - 1) - e^{-Q^2}\operatorname{erf}(Q)\}\right], \tag{31}$$

where

$$P = \sqrt{(Re_D e^{\pi\xi_\infty})/2}\,\sin(\pi\eta/2), \tag{32}$$

$$Q = \sqrt{(Re_D e^{\pi\xi_\infty})/2}\,\cos(\pi\eta/2). \tag{33}$$

It is important to note that these formulae depend on the drag coefficient C_D. Here the drag coefficient is evaluated at the cylinder surface and the following formula can be derived:

$$C_D = \frac{4}{Re_D}\int_0^1\left(\frac{\partial\omega'}{\partial\xi}\right)_0\sin(\pi\eta)\,d\eta,$$
$$-\frac{4\pi}{Re_D}\int_0^1\omega'(0,\eta)\sin(\pi\eta)\,d\eta. \tag{34}$$

Equations (30) and (31) have also been used by Keller and Takami [39], and Takami and Keller [27], and parts of them by Nieuwstadt and Keller [29] in studies concerning only the flow fields. In those works, the importance the conditions (30) and (31) also was pointed out.

For the temperature field the following procedure is used.

As $\xi \to \infty$, the streamfunction Ψ' approaches the uniform flow solution given in equation (26). If this expression of the streamfunction is inserted into equation (18) we obtain:

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2} = \frac{Re_D Pr\,\pi\,e^{\pi\xi}}{2} \times\left\{\frac{\partial T}{\partial \eta}\sin\pi\eta - \frac{\partial T}{\partial \xi}\cos\pi\eta\right\}. \tag{35}$$

The solution of equation (35) is

$$T(\xi,\eta) = e^{X\cos\pi(1-\eta)}\sum_{n=0}^{\infty}A_n K_n(X)\cos n\pi(1-\eta), \tag{36}$$

with

$$X = \frac{Re_D Pr}{4} e^{\pi\xi}.$$

The functions $K_n(X)$ are the modified Bessel functions of the second kind and A_n are arbitrary constants. As $\xi \to \infty$, we have for $K_n(X)$:

$$K_n(X) \sim (\pi/2X)^{0.5} e^{-X} \quad (37)$$

regardless of n. Equation (36) can then be written as:

$$T(\xi,\eta) \sim F(\eta) e^{X(\cos\pi(1-\eta)-1)-(\pi\xi)/2}. \quad (38)$$

If the constants A_n are known, then the function $F(\eta)$ can be determined. However, it is not necessary to know the function $F(\eta)$ explicitly since we just wish to use equation (38) as the outer boundary condition for the temperature field.

Let ξ_∞ denote the outermost boundary in the calculation domain and ξ_m a boundary situated just to the inside of ξ_∞. From equation (38) we then obtain:

$$\frac{T(\xi_\infty,\eta)}{T(\xi_m,\eta)} = \exp[(\cos\pi(1-\eta)-1)\{X_\infty - X_m\} - (\xi_\infty - \xi_m)\pi/2]. \quad (39)$$

Equation (39) is the corrected boundary condition of the temperature field to be used instead of equation (28).

An essentially the same procedure for handling the temperature boundary condition was used by Dennis *et al.* [31]. However, equation (39) was applied in a slightly different manner. Dennis *et al.* studied only pure forced convection but did not use flow fields where the asymptotic solutions (30) and (31) had been applied.

To the author's knowledge, this is the first study where both the flow and temperature conditions at a finite distance from the cylinder surface have been treated in an accurate way by matching to asymptotic solutions.

HEAT TRANSFER COEFFICIENT

Of great importance in heat transfer calculation is the film heat transfer coefficient or the Nusselt number at the interface between the fluid and the solid. Here we define the heat transfer coefficient with respect to the temperature difference $t_c - t_\infty$. The result is

$$Nu_D = \frac{hD}{k_f} = \frac{2}{\pi}\left(\frac{\partial T}{\partial \xi}\right)_0. \quad (40)$$

NUMERICAL SOLUTION PROCEDURE

The full numerical details will not be given in this report but can be found in [30] and [40]. Here only some principles will be mentioned.

The equations are solved by using second-order finite-difference approximations. On the (ξ,η)-plane a uniform grid with the same spacing in both the coordinate directions is placed. In the physical plane, we get smaller meshes near the surface of the cylinder and bigger ones far from the body.

The finite-difference equations of Ψ' and ω' are solved by using the relaxation method. However, the relaxation parameters must be smaller than unity. With increasing Reynolds number these relaxation parameters must be diminished, especially that of the vorticity equation.

If we study equations (18) and (19), we see that we can use one and the same equation (equation (18)) for the whole heat transfer field (except at the interfaces) by letting $\Psi' = 0$ within the solid since we have no flow within the solid.

The finite-difference solution of equation (18) is also obtained by the relaxation method. Within the solid layers equation (18) reduces to the Laplacian equation and then an optimal relaxation parameter $R = 2/(1+\sin\pi H)$ can be used. For the convective heat transfer field, the relaxation parameter is dependent on the value of $Re_D Pr$. For a given Reynolds number this relaxation parameter must be much smaller than unity for fluids with high Prandtl-number while for low Prandtl-number fluids a relaxation parameter larger than unity can be used.

The interfaces are treated in a special way. By Taylor-expansion of the temperature and by application of the conditions of equality in temperature and heat flux at the interfaces we obtain:

At the ith solid–solid interface:

$$T(\xi_i,\eta) = \frac{T(\xi_i - H,\eta) + (k_{s_{i+1}}/k_{s_i})T(\xi_i + H,\eta)}{1 + k_{s_{i+1}}/k_{s_i}}. \quad (41)$$

At the fluid–solid interface:

$$T(0,\eta) = \frac{T(-H,\eta) + (k_f/k_{s_n})T(H,\eta)}{1 + k_f/k_{s_n}}. \quad (42)$$

By using the corrected boundary conditions, equations (30), (31), and (39), the numerical solution should ideally be independent of the value of $(r/a)_\infty$ or ξ_∞. The values of $(r/a)_\infty$ were set equal to 14, 16.9, and 20.4. It was found that the drag coefficient was slightly dependent on $(r/a)_\infty$ while the effect of $(r/a)_\infty$ on the heat transfer coefficient was negligible.

All the calculations have been done on IBM 360/65 and 370/148 in single precision.

The influence of the step size on the numerical results was investigated by letting $H = 0.04$, 0.02, and 0.01. For the flow field, the results with $H = 0.04$ differed from those with $H = 0.02$ by one per cent, while for the heat transfer calculations the corresponding difference was about three per cent.

If computations in double precision were used $H = 0.02$ and $H = 0.01$ yielded the same heat transfer results (to at least six figures) while with computations in single precision only three figures were identical. This is due to the fact that in single precision the computer has a mantissa of only ~ 7.2 figures. In double precision the mantissa is of ~ 16.8 figures. As an effect, in single precision, the number of significant

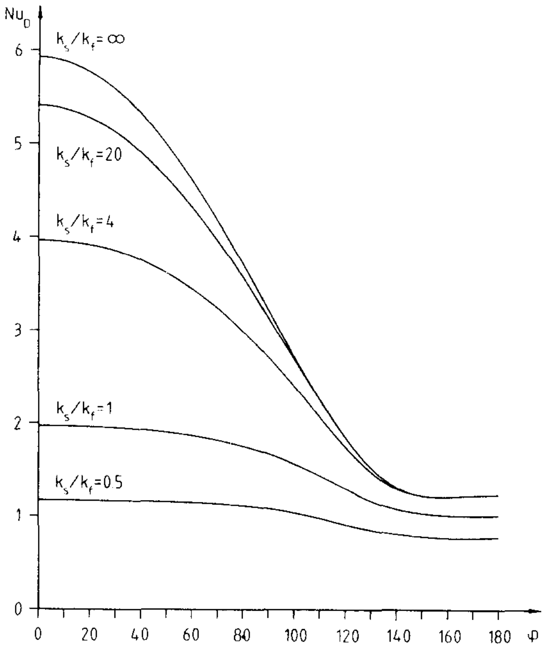

FIG. 3. Influence of the ratio k_s/k_f on Nu_D. $Re_D = 40$, $Pr = 0.72$, $D_c/D = 0.5$.

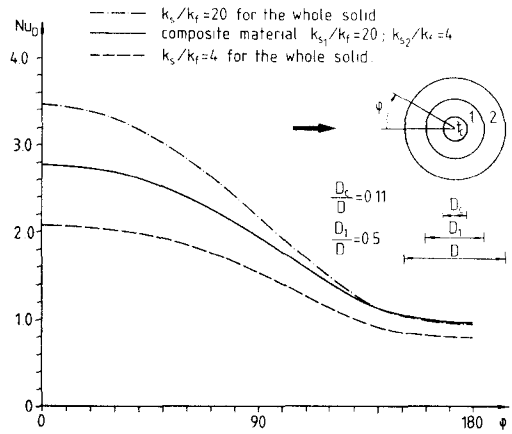

FIG. 5. Distribution of the Nusselt number along the fluid–solid interface for a two-layer composite material. $Re_D = 20$, $Pr = 0.72$.

figures in the calculated results will be very small if a very large number of operations and grid points were used.

Since the numerical calculations were done in single precision (essentially due to economical reasons and much enlarged storage requirements for double precision) the step size was chosen to $H = 0.02$.

The iterative procedure for the flow field calculations was terminated when the relative increase or decrease in the drag coefficient was smaller than 10^{-5}. For the heat transfer calculations, the iterations were terminated when the relative increases or decreases in the local Nusselt numbers and the interface temperatures were smaller than 10^{-6}.

RESULTS AND DISCUSSION

The results of the flow field calculations are in good agreement with experiments and with published numerical results but will not be given in this report. Instead the most interesting heat transfer results will be given.

In Fig. 3, the results for the local heat transfer coefficient Nu_D are given for $Re_D = 40$ and $D_c/D = 0.5$. The solid material has here a uniform thermal conductivity. The ratio k_s/k_f appears as a parameter. As is evident, the heat transfer coefficient is greatly influenced by k_s/k_f. As k_s/k_f is decreased, the local Nusselt number is decreased especially on the forward side of the cylinder surface. This is because of the increased insulation obtained for $k_s/k_f < \infty$. However, it is also clear that k_s/k_f has to be rather small if the heat transfer coefficients for $k_s/k_f = \infty$ (equal to the isothermal case) should be changed. Figure 4 shows the corresponding surface temperature distributions. As k_s/k_f is decreased the temperature difference between the forward and backward stagnation points is increased.

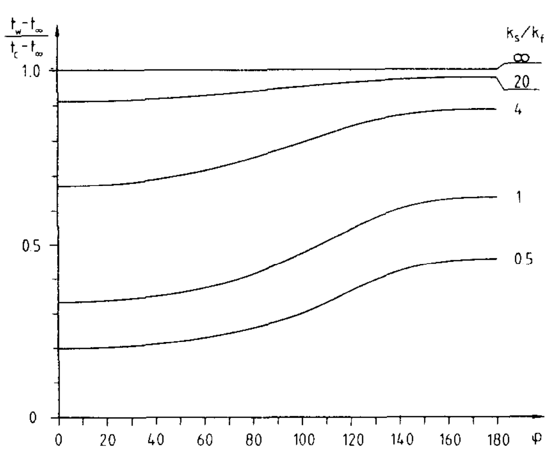

FIG. 4. Surface temperature distribution. Influence of the ratio k_s/k_f. $Re_D = 40$, $Pr = 0.72$, $D_c/D = 0.5$.

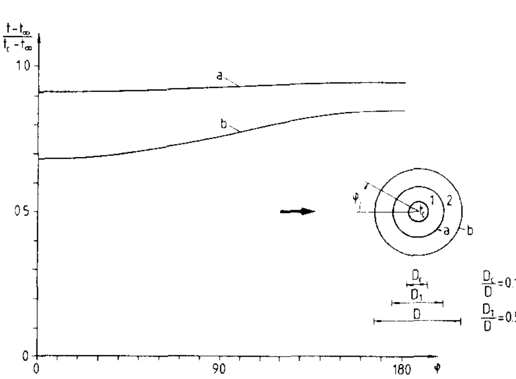

FIG. 6. Interface temperature distributions. $Re_D = 20$, $Pr = 0.72$, $k_{s_1}/k_f = 20$, $k_{s_2}/k_f = 4$.

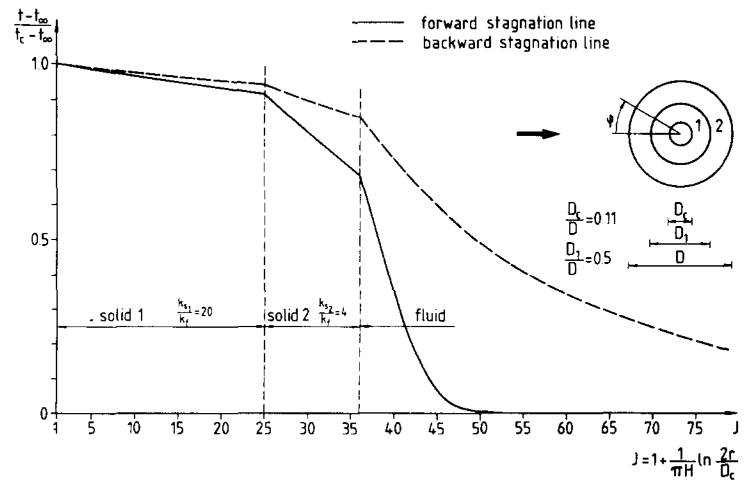

FIG. 7. Temperature distribution along the forward and backward stagnation line. Two-layer composite material. $Re_D = 20$, $Pr = 0.72$.

Figure 5 shows the distribution of the Nusselt number along the cylinder surface for a two-layer composite material. The Reynolds number is $Re_D = 20$ and the diameter ratios are $D_c/D = 0.11$ and $D_1/D = 0.5$. The results are given for $k_{s_1}/k_f = 20$ and $k_{s_2}/k_f = 4$. Comparison is made with the cases when the whole solid has $k_s/k_f = 20$ and $k_s/k_f = 4$, respectively. Again, the remarkable effect of the thermal conductivity is clear.

The corresponding interface temperature distributions are shown in Fig. 6. For the solid–solid interface the temperature difference between the points $\varphi = 180°$ and $\varphi = 0°$ is rather small. This means that in the solid layer closest to the core region, the heat transfer is almost due to radial conduction. The reason for this might be that the core region has the strongest influence on the heat transfer in the first solid layer.

Figure 7 shows the temperature distributions along the forward and backward stagnation lines. Due to the differences in the thermal conductivity between the solids and the fluid, discontinuities in the temperature gradients appear at the interfaces. For the low Reynolds numbers considered in this work, the activity in the wake region is low and as is evident from Figs. 3 and 5, the forward side of the cylinder dominates the heat transfer. As is shown in Fig. 7, this results in great differences between the temperature distributions along the forward and backward stagnation lines.

In Fig. 8, the influence of the Prandtl-number on the local Nusselt number can be studied. The Reynolds number is $Re_D = 40$ and the solid material has a uniform thermal conductivity with $k_s/k_f = 4$. The diameter ratio is $D_c/D = 0.11$. For mercury ($Pr = 0.021$), the variation in the Nusselt number along the cylinder surface is small. This is so because for small Pr-number, the convection terms in equation (18) no longer dominate the heat transfer within the fluid, that is the heat conduction in the fluid becomes

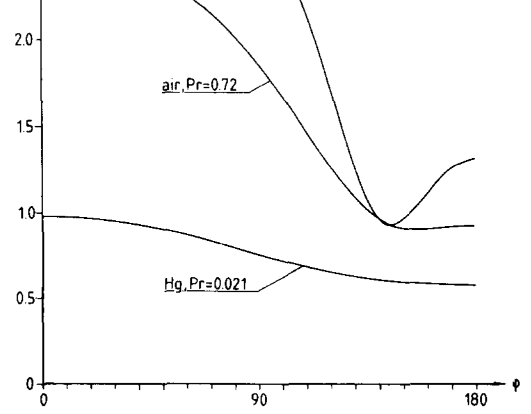

FIG. 8. Influence of the cooling medium on the local heat transfer coefficient. $Re_D = 40$, $D_c/D = 0.11$, $k_s/k_f = 4$.

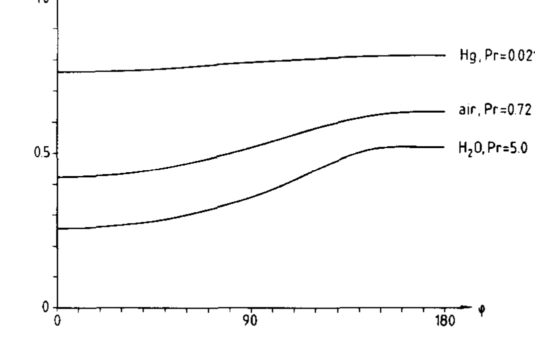

Fig. 9. Influence of the cooling medium on the surface temperature distribution. $Re_D = 40$, $D_c/D = 0.11$, $k_s/k_f = 4$.

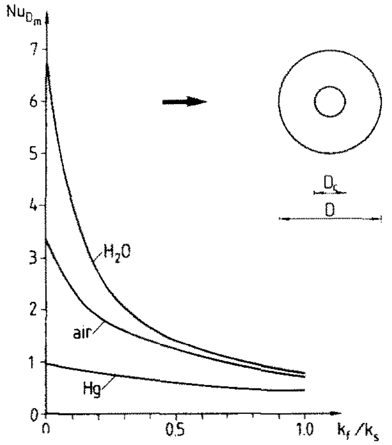

Fig. 10. Influence of k_f/k_s and Pr on the mean heat transfer coefficient. $Re_D = 40$, $D_c/D = 0.11$.

very important. On the other hand, for high Prandtl-number (for instance water $Pr = 5$) the convection terms become very strong as is shown in Fig. 8. Also the contribution from the wake region becomes larger when the Prandtl-number is increased.

Figure 9 shows the corresponding surface temperature distributions.

The influence of the thermal conductivity ratio k_f/k_s and the Prandtl-number on the mean heat transfer coefficient is shown in Fig. 10 for $Re_D = 40$ and $D_c/D = 0.11$. From this figure it is clear that for higher Prandtl-number, the mean heat transfer coefficient is more sensitive to the value of k_f/k_s.

The effect of D_c/D on the heat transfer results can be studied by comparing the results in Figs. 3 and 8. For air ($Pr = 0.72$) and $k_s/k_f = 4$, we see that for $D_c/D = 0.11$ the local heat transfer coefficients are smaller than those for $D_c/D = 0.5$. This is so because for $D_c/D = 0.11$ the solid insulation layer is thicker than that for $D_c/D = 0.5$.

For $D_c/D = 1$ or $k_s/k_f = \infty$, the mean heat transfer coefficients were compared with the well-known empirical formulas available in the literature. It was found that the formula due to Kramers [41] gave mean heat transfer coefficients with a deviation from the numerical results calculated in this work within 8%. ($5 \leq Re_D \leq 40$). The formula from Collis and Williams [42] gave results with a deviation within 5%.

CONCLUSION

The conjugated heat transfer problem for a circular cylinder with a heated core region in low Reynolds number flow has been studied. The ratio of the thermal conductivities (k_s/k_f) has been found to greatly influence the heat transfer. High Pr-number fluids have been shown to be more sensitive to the thermal conductivity ratio than low Pr-number fluids.

The condition within the core region, $t = t_c$, can without any large effort be changed to an arbitrary temperature distribution or to an arbitrary heat flux distribution.

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TRANSFERTS THERMIQUES CONJUGUES AUTOUR DE CYLINDRES CIRCULAIRES AUX FAIBLES NOMBRES DE REYNOLDS

Résumé—On considère le problème du transfert thermique conjugué à partir d'un cylindre circulaire avec un noyau central chauffé, aux faibles nombres de Reynolds. Le solide est constitué de plusieurs couches avec différentes conductivités thermiques. Les équations d'énergie du fluide et du solide sont résolues sous la condition d'égalité de la température et du flux de chaleur aux interfaces fluide–solide et solide–solide. Des conditions à l'infini sont appliquées pour le champ d'écoulement par la solution asymptotique d'Imai et, pour le champ de température, par la solution asymptotique correspondant aux températures. Des résultats sont donne's pour différentes matériaux solides et fluides.

GLEICHZEITIGES AUFTRETEN VON FREIER UND ERZWUNGENER KONVEKTION AN KREISZYLINDERN IN STRÖMUNGEN BEI NIEDRIGEN REYNOLDS-ZAHLEN

Zusammenfassung—Es wird das Problem der gleichzeitig auftretenden freien und erzwungenen Konvektion an einem im Kern beheizten Zylinder in Strömungen mit niedrigen Reynolds-Zahlen untersucht. Der Festkörper soll aus mehreren Schichten mit unterschiedlicher Wärmeleitfähigkeit bestehen. Die Energiegleichungen für die Flüssigkeit und den Festkörper werden unter der Bedingung gleicher Wärmestromdichte und der gleichen Temperatur an der Grenzfläche flüssig-fest und den Grenzflächen fest-fest gelöst. Die Unendlichkeitsbedingungen werden auf das Strömungsfeld durch die asymptotische Lösung nach Imai und auf das Temperaturfeld durch eine Anpassung an die entsprechende asymptotische Lösung für die Temperatur angewendet. Es werden Resultate für verschiedene Festkörpermaterialien und Flüssigkeiten angegeben.

СОПРЯЖЕННАЯ ЗАДАЧА ТЕПЛОПЕРЕНОСА ОТ КРУГЛОГО ЦИЛИНДРА В ПОТОКЕ С МАЛЫМИ ЧИСЛАМИ РЕЙНОЛЬДСА

Аннотация — Рассматривается сопряженная задача теплопереноса от нагреваемого изнутри кругового сплошного цилиндра при обтекании его потоком жидкости в случае малых чисел Рейнольдса. Цилиндр может состоять из нескольких слоев с различной теплопроводностью. Уравнение энергии для жидкости и уравнения теплопроводности для цилиндра решаются в предположении равенства плотностей теплового потока и температур на границах раздела жидкость — твердое тело и твердое тело–твердое тело. Условия на бесконечности для поля скоростей определяются асимптотическим решением, предложенным Имаи, а для поля температур — на основании соответствующего асимптотического решения. Исследованы различные твердые материалы и жидкости.